**Parallel and perpendicular lines**

 **A LEVEL LINKS**

 **Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

* When lines are parallel they have the same gradient.
* A line perpendicular to the line with equation *y* = *mx* + *c* has gradient .

Examples

**Example 1** Find the equation of the line parallel to *y* = 2*x* + 4 which passes through
the point (4, 9).

|  |  |
| --- | --- |
| *y* = 2*x* + 4*m* = 2*y* = 2*x* + *c*9 = 2 × 4 + *c*9 = 8 + *c**c* = 1*y* = 2*x* + 1 | **1** As the lines are parallel they have the same gradient.**2** Substitute *m* = 2 into the equation of a straight line *y* = *mx* + *c*.**3** Substitute the coordinates into the equation *y* = 2*x* + *c***4** Simplify and solve the equation.**5** Substitute *c* = 1 into the equation *y*= 2*x* + *c* |

**Example 2** Find the equation of the line perpendicular to *y* = 2*x* − 3 which passes through
the point (−2, 5).

|  |  |
| --- | --- |
| *y* = 2*x* − 3*m* = 25 = 1 + *c**c* = 4 | **1** As the lines are perpendicular, the gradient of the perpendicular line is .**2** Substitute *m* =  into *y* = *mx* + *c*.**3** Substitute the coordinates (–2, 5) into the equation **4** Simplify and solve the equation.**5** Substitute *c* = 4 into . |

**Example 3** A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through
its midpoint.

|  |  |
| --- | --- |
| , ,  and  Midpoint =   | **1** Substitute the coordinates into the equation  to work out the gradient of the line.**2** As the lines are perpendicular, the gradient of the perpendicular line is .**3** Substitute the gradient into the equation *y* = *mx* + *c*.**4** Work out the coordinates of the midpoint of the line.**5** Substitute the coordinates of the midpoint into the equation.**6** Simplify and solve the equation.**7** Substitute  into the equation . |

Practice

**1** Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

 **a** *y* = 3*x* + 1 (3, 2) **b** *y* = 3 – 2*x* (1, 3)

 **c** 2*x* + 4*y* + 3 = 0 (6, –3) **d** 2*y* –3*x* + 2 = 0 (8, 20)

**2** Find the equation of the line perpendicular to *y* = *x* – 3 which passes through the point (–5, 3).

**Hint**

If *m* =  then the negative reciprocal 

**3** Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

 **a** *y* = 2*x* – 6 (4, 0) **b** *y* = *x* +  (2, 13)

 **c** *x* –4*y* – 4 = 0 (5, 15) **d** 5*y* + 2*x* – 5 = 0 (6, 7)

**4** In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

 **a** (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

**5** Work out whether these pairs of lines are parallel, perpendicular or neither.

 **a** *y* = 2*x* + 3 **b** *y* = 3*x* **c** *y* = 4*x* – 3
 *y* = 2*x* – 7 2*x + y* – 3 = 0 4*y* + *x* = 2

 **d** 3*x* – *y* + 5 = 0 **e** 2*x* + 5*y* – 1 = 0 **f** 2*x* – *y* = 6

 *x* + 3*y* = 1 *y* = 2*x* + 7 6*x* – 3*y* + 3 = 0

**6** The straight line **L1** passes through the points *A* and *B* with coordinates (–4, 4) and (2, 1), respectively.

 **a** Find the equation of **L1** in the form *ax* + *by* + *c* = 0

 The line **L2** is parallel to the line **L1** and passes through the point *C* with coordinates (–8, 3).

 **b** Find the equation of **L2** in the form *ax* + *by* + *c* = 0

 The line **L3** is perpendicular to the line **L1** and passes through the origin.

 **c** Find an equation of **L3**

Answers

**1 a** *y* = 3*x* –7 **b** *y* = –2*x* + 5

 **c** *y* = –*x* **d** *y* = *x* + 8

**2** *y* = −2*x* – 7

**3** **a** *y* = –*x* + 2 **b** *y* = 3*x* + 7

 **c** *y* = –4*x* + 35 **d** *y* = *x* – 8

**4** **a** *y* = –*x* **b** *y* = 2*x*

**5** **a** Parallel **b** Neither **c** Perpendicular

 **d** Perpendicular **e** Neither **f** Parallel

**6** **a** *x* + 2*y* – 4 = 0 **b** *x* + 2*y* + 2 = 0 **c** *y* = 2*x*